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AN EXPERIMENT AND ANALYSES ON ELASTIC WAVES IN BEAMS FROM LATERAL IMPACT†

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Abstract—A long beam is laterally loaded with a spatially uniform impulse over a length equal to three beam thicknesses. Loading is produced by short-duration magnetic pressure pulses and the response is measured with strain gages. Strain measurements are compared with predictions from beam theories and a two-dimensional numerical analysis.

INTRODUCTION

Many investigations associated with the transient response of beams have appeared in the literature and a recent survey is contained in the report by Colton[1]. In this paper, a long aluminum beam is laterally loaded with a spatially uniform impulse over a length equal to three beam thicknesses. Loading is produced by a short-duration ($\sim 4\mu$s) pressure pulse. Pulse duration is nearly equal to the time it takes a disturbance traveling at the bar velocity of aluminum to travel one beam thickness (19·05 mm, 0·75 in). Response is measured with strain gages mounted on both beam surfaces at eight beam thicknesses from the center of the applied impulse. Predicted strain-time histories are calculated by employing previously derived solutions for point impulse loads on Timoshenko[2] and elementary[3] beams and the two-dimensional wave propagation code, TOODY[4, 5]. Strain measurements indicate that some high-frequency bending and axial strain components are not predicted by the Timoshenko beam theory; however, the measured strain components are in general agreement with predictions from the TOODY computer program.

Accuracy of the TOODY code for calculating elastic and elastic–plastic wave motions has been recently demonstrated for problems in which the response was dominated by axial motion[5]. This problem offers another test for TOODY in which the response is dominated by flexural motion.

EXPERIMENT

A sketch of the experimental arrangement is shown in Fig. 1. The center of the beam is placed over the center of two thin 1100-0 aluminum conductors which are three times as wide as the beam thickness. The conductors, which are insulated from each other and the beam, are connected to a fast-discharge capacitor bank. When the switch is closed, a magnetic pressure is

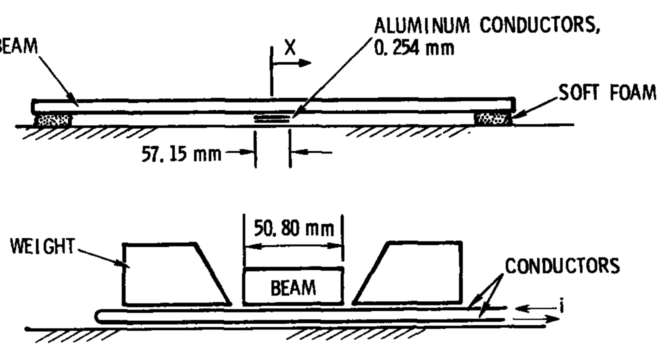

Fig. 1. Experimental arrangement.

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generated between the conductors which loads the beam with a spatially uniform† short-duration pressure pulse over the conductor width. Two beam experiments were conducted and produced nearly identical data. For brevity, results from only one experiment are presented.‡

Specimen. A 1·52 m (5 ft) long beam was cut from a 19·05 by 50·8 mm (3/4 by 2 in) bar of 6061-T6511 aluminum.

Magnetic pressure pulse. The pressure generated between two closely spaced parallel conductors is

$$p(t) = (\mu/2)[i(t)/b]^2 \quad (1)$$

where i is the current from capacitor discharge, b is the conductor width, and μ is the permeability of space. A sine current-time pulse is produced by the shaping technique described in [6] and the beam is loaded by a sine-squared pressure pulse. Current is measured with a Rogowski loop and is shown in Fig. 2; the corresponding impulse per unit area is 55 $Pa \cdot s$.

Impulse magnitudes were also determined from two auxiliary experiments. A short bar with length equal to the conductor width b (57·15 mm, 2·25 in) was cut from the end of the beam specimen. The unrestrained bar was impulse loaded uniformly along its length with the same procedure as the long beam. The vertical distance traveled by the short bar at $t = 50$ and 100 ms was measured with the multiflash photographic method[7]. Impulses inferred from the current trace and rigid body translation were 57 and 58 $Pa \cdot s$, respectively, for one experiment; and 66 and 67 $Pa \cdot s$, respectively, for the other. Thus, the current and translational measurements are in close agreement.

Strain measurement. Strain responses were measured with two Micro-Measurements EA-06-125AC-350 strain gages mounted to the beam surfaces at eight thicknesses (152·4 mm, 6·0 in) from the center of the conductors. The strain-time histories for the top gage ϵ_t and bottom gage ϵ_b are shown in Figs. 3a, b. The bending strain $\frac{1}{2}(\epsilon_t - \epsilon_b)$, shown in Fig. 4,§ was obtained by electrically subtracting the top and bottom signals; and the axial strain $\frac{1}{2}(\epsilon_t + \epsilon_b)$, shown in Fig. 5a, was obtained by electrically adding the signals.

THEORETICAL PREDICTIONS

Beam theories. Strain-time histories were calculated by employing the solutions for point impulse loads on Timoshenko[2] and elementary[3] beams. Predicted bending strain-time

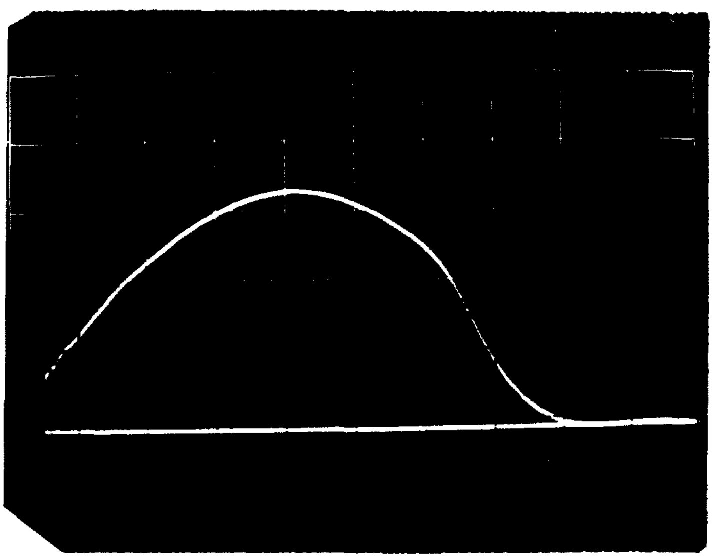
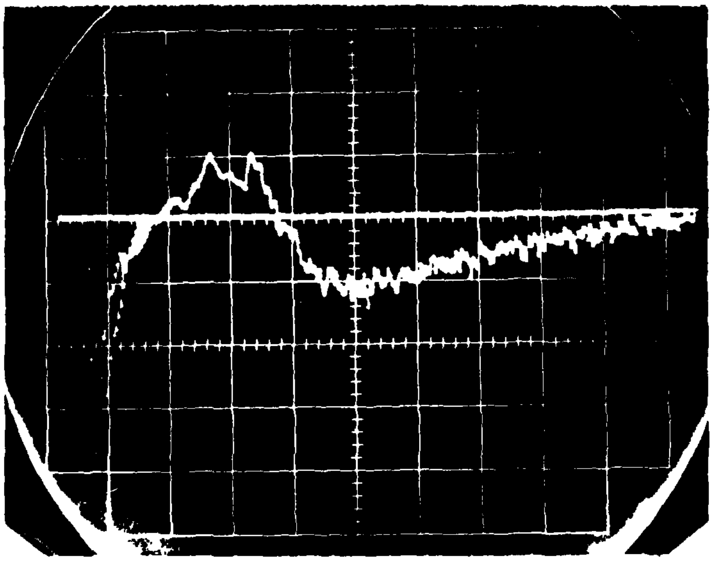


Fig. 2. Current-time; 111·22 ka/div, 0·5 μ s/div.

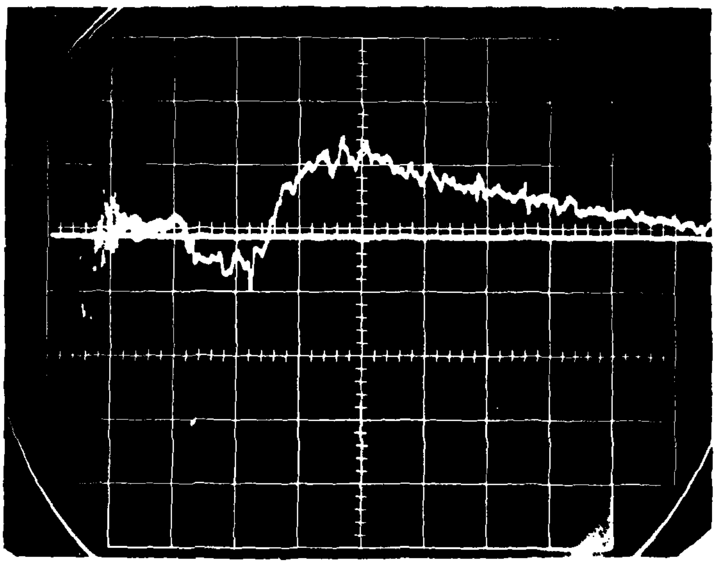
†Spatial uniformity depends on separation distance between conductors; see [8]. For this experiment the conductors are insulated with 0·003 in Mylar and it is reasonable to assume a uniform pressure distribution.

‡Traces from the other experiment are available from the author (M.J.F.).

§For the first ~30 μs, the signals are recovering from electrical noise induced by capacitor discharge. At ~30 μs the signal returns to the base line which, as shown in Fig. 4, is prior to strain response.



(a)



(b)

Fig. 3. Strain-time: (a) top surface strain ϵ_t; (b) bottom surface strain ϵ_b; 100 $\mu\epsilon$/div, 20 μs/div.

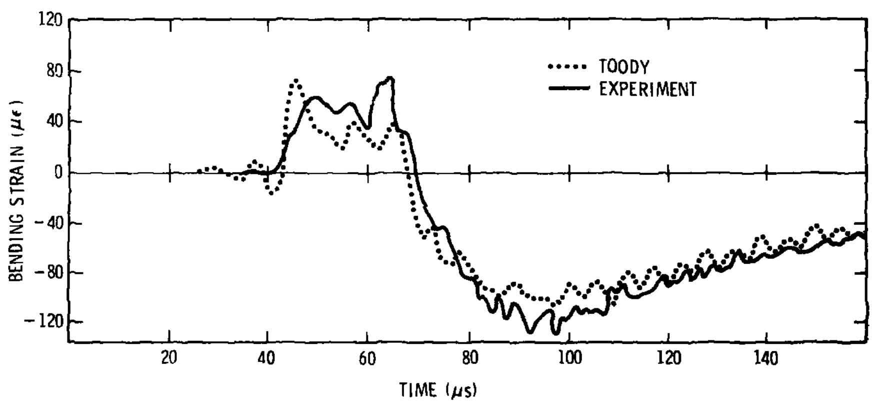
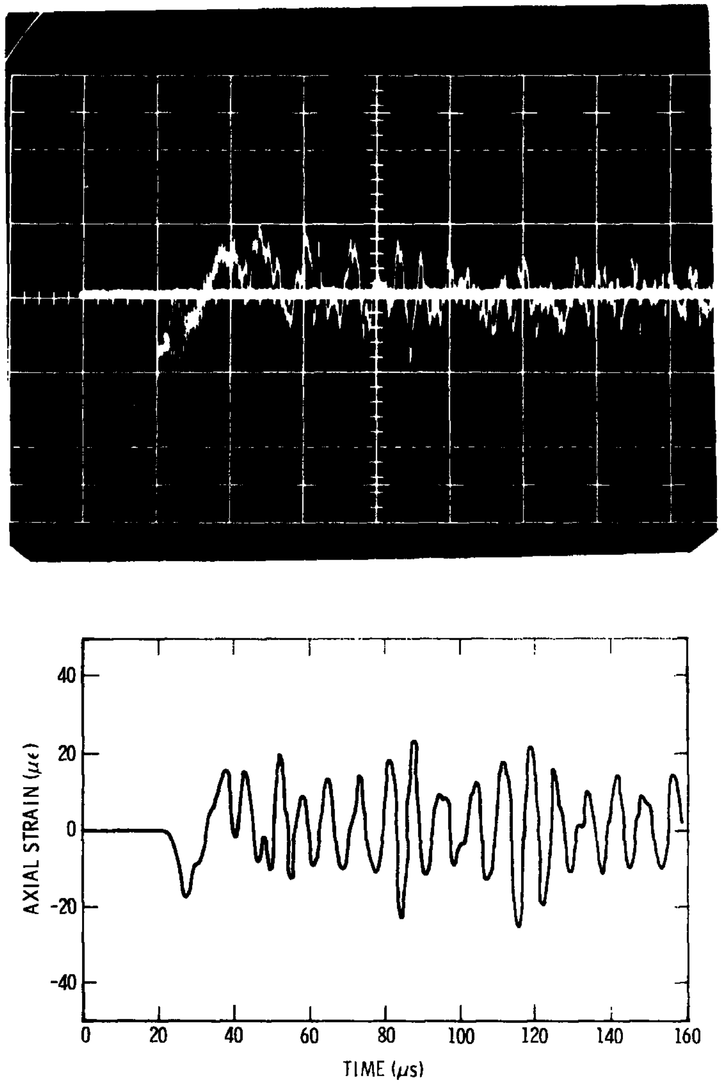


Fig. 4. Bending strain-time. $\frac{1}{2}(\epsilon_t - \epsilon_b)$.



(b)

Fig. 5. Axial strain-time, $\frac{1}{2}(\epsilon_t + \epsilon_b)$: (a) 25 $\mu\epsilon$/div, 20 μs/div; (b) TOODY predictions.

histories at 152·4 mm (6·0 in) from the center of loading are shown in Fig. 6. The uniform spatial distribution was modeled by sixteen point loads with a 4 μsec duration sine-squared time distribution.

Two-dimensional analysis. The TOODY [4] two-dimensional elastic–plastic wave propagation computer code was used to numerically simulate the experiment. TOODY is a second-order accurate, artificial viscosity, finite-difference code which solves the equations of motion in Lagrangian coordinates. For problems involving two spatial coordinates (e.g. plane strain) the code solves numerically the finite-difference approximations to the exact two-dimensional equations. However, additional approximations are required for application of the code to problems of plane stress or other problems of three-dimensional motion.

Wave motions in the beam illustrated in the experimental arrangement shown in Fig. 1 are approximated as a problem in plane stress. The plane stress approximation for TOODY code calculations is accomplished by modifying the elastic constants of the aluminum beam and performing a plane strain calculation. Shear modulus and propagation velocity are the same for plane stress and plane strain. However, the bending wave velocities are $C_b^2 = E/\rho$ for plane stress and $C_b^2 = E/[(1-\nu^2)\rho]$ for plane strain. To simulate the plane stress assumption and perform plane strain calculations, Young's modulus $E = 68{\cdot}9$ GPa (10×10^6 psi) and Poisson's ratio $\nu = \frac{1}{3}$ for aluminum are taken as $\bar{E} = 64{\cdot}6$ GPa and $\bar{\nu} = \frac{1}{4}$ for input to TOODY. The modified material properties have the same shear modulus as aluminum and the same bending wave

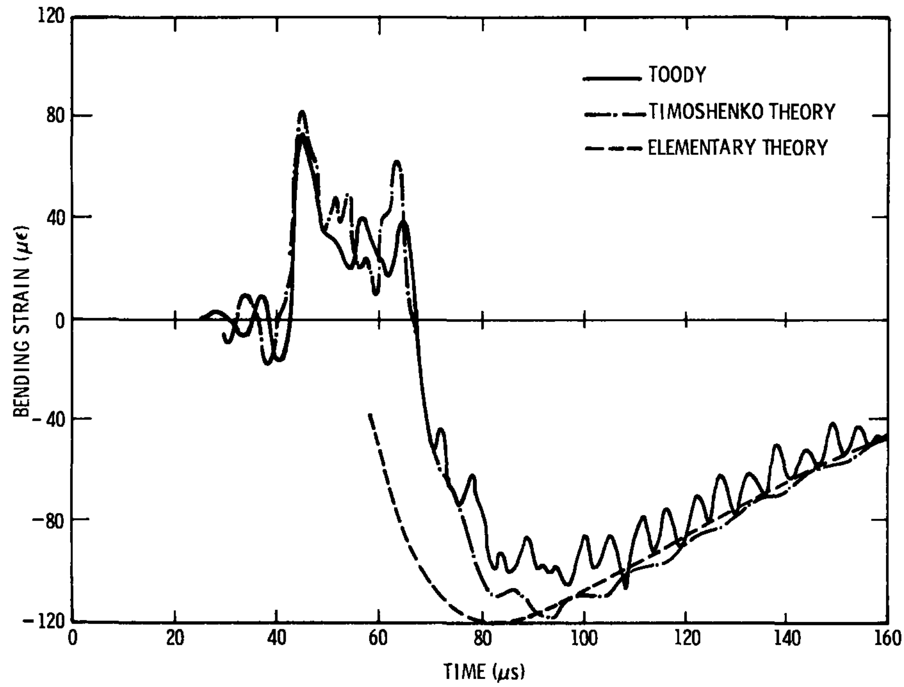


Fig. 6. Predicted bending strain-time.

velocity as aluminum in a state of plane stress. Results of the code calculations are presented in Figs. 4–6.

DISCUSSION

Accuracy of the TOODY Code for calculating elastic and elastic-plastic wave motions has been recently demonstrated for problems in which the response was dominated by axial motion[5]. The problem presented in this paper offers another test for TOODY in which the response is dominated by flexural motion. Bending strain, $\frac{1}{2}(\epsilon_t - \epsilon_b)$, predicted by TOODY is compared with Timoshenko and elementary beam theories in Fig. 6. Predictions from elementary beam theory do not include the effects of transverse shear and rotary inertia and are not in agreement with the Timoshenko beam calculation until after $\sim 90\,\mu s$. Timoshenko beam calculations are in general agreement with TOODY except for the higher frequency oscillations predicted by TOODY.

As shown in Fig. 4, TOODY bending strain predictions are in good agreement with the measured strain. The axial strain components are shown in Fig. 5. A direct comparison of measurement and prediction for axial strain is not provided because of the high frequency response detail; however, general agreement between measurement and TOODY can be observed in Fig. 5.

In summary, strain measurements from a laterally impulse-loaded beam are compared with beam theories and a two-dimensional numerical analysis. Close agreement is shown between strain measurements and two-dimensional code calculations.

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